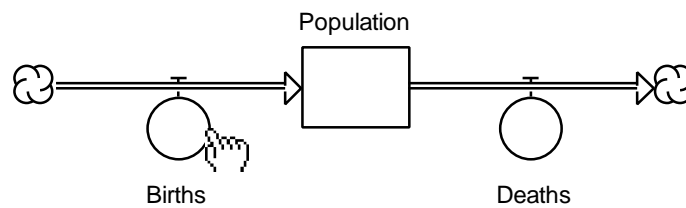


# Modeling Exercises

## Section 2



Prepared for the  
MIT System Dynamics in Education Project  
Under the Supervision of  
Prof. Jay W. Forrester

by  
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August, 1994

Vensim Examples added October 2001

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## 1. ABSTRACT

This paper is the second in a continuing series of papers designed to enhance the reader's modeling skills. Modeling Exercises Section 2 contains two separate modeling exercises. In the first exercise the reader will build a system dynamics model based on a detailed description of an urban system. The reader will run the urban model under varying conditions. The second exercise asks the reader to perform mental simulations on a pre-built model, and to estimate the model behavior under different conditions. The reader is expected to have prior experience with system dynamics, model building, and mental simulation. Before beginning this paper, please review Modeling Exercises Section 1, which outlines the process of model building.<sup>1</sup>

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<sup>1</sup> Joseph G. Whelan, 1994. *Modeling Exercises Section 1* (D-4421), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May, 1994, 37 pp.

Mental simulation is explained in the following papers:

Joseph G. Whelan, 1995. *Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback* (D-4487), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July 27, 15 pp.

Helen Zhu, 1996. *Beginner Modeling Exercises Section 3: Mental Simulation of Simple Negative Feedback* (D-4536), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, January 28, 23 pp.

Alan Coronado, 1996. *Beginner Modeling Exercises Section 4: Mental Simulation: Adding Constant Flows* (D-4546), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 27 pp.

Laughton Stanley and Helen Zhu, 1996. *Beginner Modeling Exercises Section 5: Mental Simulation of Combining Feedbacks in First Order Systems* (D-4593), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 28 pp.

## 2. INTRODUCTION

Modeling is a crucial element of system dynamics. People rely on mental models daily to make decisions without even questioning the model being used. Creating and simulating models using STELLA or other modeling software<sup>2</sup> allows the modeler to solidify his mental models. The modeler thus observes the behavior of complex systems where mental models are inadequate. Both mental and computer models can give wrong answers. A computer will give an answer with precision even if that answer is based on an inadequate model. A computer will not say “A population cannot be negative,” or “The number of sick people cannot be greater than the total population.” The modeler must be the one to verify the quality of a model. This paper and future Modeling Exercises in Road Maps will enhance the reader’s modeling skills.

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<sup>2</sup> Powersim or Vensim can be used if the reader can make interpretations from the material in this paper.

### 3. EXERCISE 1: URBAN DYNAMICS<sup>3</sup>

In this exercise the reader will build a simple model to explain growth and stagnation in an urban area. Read the material in Section 3.1, then answer the questions in Section 3.2. Section 3.3 further describes the system, and Section 3.4 gives instructions to build a STELLA model to fit the description. The model should then be used to answer the questions given in Section 3.5.

#### 3.1 Background of the Urban System

A city's life cycle is characterized by a period of economic growth followed by a period of transition toward equilibrium. During the growth period, the city appears to be economically healthy: business activity is expanding and unemployment is low. The "Population" and the number of "Business Structures" grow quickly. During the transition period, conditions become less desirable and pressures arise which impede further growth. The "Construction" rate of "Business Structures" becomes smaller, and so does the rate of "In Migration" into the city. In equilibrium, when the "Population" and the number of "Business Structures" stop growing, the city suffers from problems such as high unemployment.

#### 3.2 Preliminary Exploration of the Urban System<sup>4</sup>

Please answer the following questions before continuing.

1. What is the problem the model will address? What is the reason for building the model? What are the key variables in the system?

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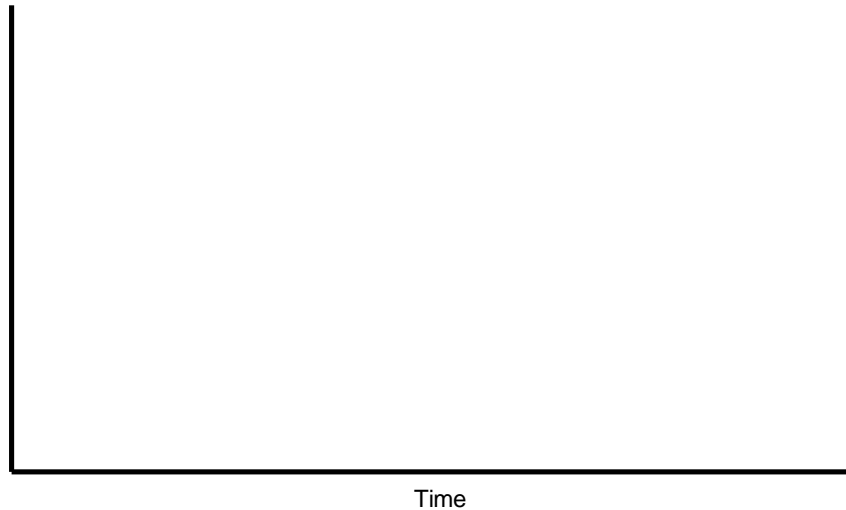
<sup>3</sup> Note: The following exercise was based on an exercise prepared by John Sterman. It has been revised for use in Road Maps. The model structure and description were adapted from the population-business (POPBSN) model in Introduction to Urban Dynamics.

John Sterman, 1981. Principles Of Systems I, 15.852/872, Fall 1981, Assignment #3: Urban Modeling Assignment (D-3296-1), System Dynamics Group, Massachusetts Institute of Technology.

Alfeld, L. E., and Graham, A. K., 1976. Introduction to Urban Dynamics. Cambridge, MA: Productivity Press. The POPBSN model is developed in Chapter 5 of the book.

<sup>4</sup> Suggested answers to this question are included in Section 5.

Sketch the expected behavior of all the key variables over an appropriate time horizon on the blank graph provided.



### 3.3 General Description of the Urban System

The availability of “Jobs” and promise of higher incomes are prime motivations for “In Migration” into an urban area. People tend to move to areas where employment opportunities are favorable. Television news, newspaper stories, advertisements, word-of-mouth accounts, and industrial recruiting all tend to create perceptions of higher wages and greater opportunities in areas where “Labor Availability” is low. On the other hand, in areas where “Labor Availability” is high, job conditions are perceived to be relatively unfavorable.<sup>5</sup> People tend to migrate to cities that, at a given time, are perceived to offer more job opportunities.

When people move into or out of an area, they add to or subtract from the number of people in the area’s “Labor Force.” Businesses cannot ignore the “Labor Availability” in their location and expansion decisions. Readily available labor allows businesses greater

flexibility in choosing employees and shortens the time necessary to fill open positions. Moreover, high “Labor Availability” tends to decrease wage competition for labor among businesses.

### **3.4 Detailed Instructions for Building the Urban Growth Model**

The urban growth model that the reader is asked to build in Section 3.5 should focus on the dynamics of population and business growth within a fixed area of a city. The model should contain two sectors: the “Business Structures” sector and the “Population” sector.

#### **3.4.1 The “Business Structures” Sector**

The “Business Structures” sector consists of a level called “Business Structures” with initial value of 1000. The number of “Business Structures” increases through a rate of “Construction” and decreases through a rate of “Demolition.”

The rate of “Demolition” is determined by the number of existing “Business Structures” and the “Average Structure Lifetime” of 50 years.

The rate of “Construction” can be modeled as a product of “Business Structures” and several factors:<sup>6</sup>

? A “Construction Fraction” represents “Construction” as 2% of “Business Structures” under normal conditions—when there is ample land, and labor availability matches the available jobs. The “Construction Fraction” thus has a value of 0.02.

? A “Land Availability Multiplier” is a nonlinear function of the “Land Fraction Occupied.” “Land Fraction Occupied” is defined as the number of “Business Structures”

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<sup>5</sup> “Labor Availability,” as it will be defined in the urban growth model, is the ratio of the “Labor Force” in the urban area to the total number of “Jobs.” When the “Labor Force” is greater than the number of “Jobs” (“Labor Availability” is greater than 1), not all the workers in the area can find “Jobs,” and the area experiences unemployment. Thus, “Labor Availability” represents the concept of unemployment from the point of view of the businesses.

multiplied by the average “Land per Structure” (1 acre per structure), and divided by the total “Land Area” of 5000 acres. When almost all “Land Area” is available, the “Land Availability Multiplier,” and thus “Construction,” is quite low because there are very few businesses around to provide capital. As more of the “Land Area” is occupied, the effect on “Construction” increases. Eventually, the “Land Area” is used up, and the “Land Availability Multiplier” and “Construction” drop again, reaching zero when all the “Land Area” is occupied. In equilibrium, “Land Availability Multiplier” is equal to 1.

? A “Labor Availability Multiplier” is a nonlinear function of “Labor Availability.” “Labor Availability” is the ratio of “Labor Force” to “Jobs.” “Labor Force” is the product of the “Population” and the “Labor Participation Fraction” of 0.35, or 35%. “Jobs” is the product of “Business Structures” and 20 “Jobs per Structure.” When “Labor Availability” is greater than 1 (“Labor Force” is greater than “Jobs”), businesses are more likely to construct new “Business Structures.” When “Labor Availability” falls below 1, the “Labor Availability Multiplier” and “Construction” fall. In equilibrium, “Labor Availability Multiplier” is equal to 1.

### ***3.4.2 The “Population” Sector***

The “Population” sector consists of a level called “Population.” The initial value of “Population” is 50000 people. “Population” increases through the rates of “Births” and “In Migration,” and decreases through the rates of “Deaths” and “Out Migration.”

The rate of “Births” is a constant percentage of the “Population.” Approximately 1.5 people are born per 100 people per year.

The rate of “Deaths” is also a constant percentage of the “Population.” The “Average Lifetime” of a person is approximately 67 years.

The rate of “Out Migration” equals 8% of the existing “Population.”

The rate of “In Migration” is also a product of “Population” and several factors:

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<sup>6</sup> In system dynamics, rates are often defined as a level multiplied by one or more factors. These factors can be aggregated into the format of a “normal” value and one or more multipliers. The normal represents the ratio of the rate to the level under standard conditions of equilibrium, while the multipliers all vary around the value of 1 to include the effects of varying conditions on the system. It is considered good practice if the multipliers are all equal to 1 in equilibrium. A later section of Road Maps will discuss how this can be achieved.

The “In Migration Normal” of 8% is the fraction of the “Population” that immigrates into the city each year under normal conditions.

The “Job Attractiveness Multiplier” depends on “Labor Availability.” When there are many available jobs (“Labor Availability” is less than 1 because “Labor Force” is smaller than “Jobs”), the “Job Attractiveness Multiplier” increases to values greater than 1. People are inclined to move to the city. When available jobs are scarce (“Labor Availability” is greater than 1 because “Labor Force” is greater than “Jobs”), the “Job Attractiveness Multiplier” decreases to values between 0 and 1. People tend not to immigrate to the city.

### **3.5 Exploration of the Urban Growth Model<sup>7</sup>**

2. State the initial working hypothesis as to why “Business Structures” and “Population” initially grow and eventually become stagnant. Use the description above for help with the answer.
  
3. Using the instructions from Section 3.4, map out in STELLA the two interacting sectors of the “Business Structures” and the “Population.”
  
4. Define the elements in the model and enter the model equations correctly into the computer. Make sure to indicate clearly any numerical assumptions not given above (e.g., values assumed in nonlinear “graphical functions”). Try to make the model clear so that others can understand its structure easily. Then simulate the model. Did the model produce a behavior similar to that predicted earlier?
  
5. What causes the growth of “Population” and “Business Structures” of an urban area during the early years of its development? Use the structure of the model to answer this question. How does the answer relate to the answer to question 2?

---

<sup>7</sup> Suggested answers to the following questions are included in Section 5.

6. What is the behavior of “Labor Availability” during the life cycle of an urban area? What causes the behavior?

7. How does the finite “Land Area” come to limit growth? Will all the available “Land Area” be filled up in equilibrium?

Use the model to show the effects of annexing surrounding land to increase the “Land Area” to 7500 acres. Initialize the model in equilibrium.<sup>8</sup> Use the STEP function in STELLA to increase the “Land Area” in year 10. How does the behavior change? Will all the “Land Area” be filled up in equilibrium?

8. Does the assumption of a fixed “Land Area” invalidate the model? Most cities can and have expanded from their original areas. How does such expansion influence the results given by the model? For example, what would be the likely consequences of expanding the “Land Area” within which the city is allowed to grow? Simulate the model with a sequence of such expansions, say every 20 years. (The reader might want to refer back to the analysis in question 7.)

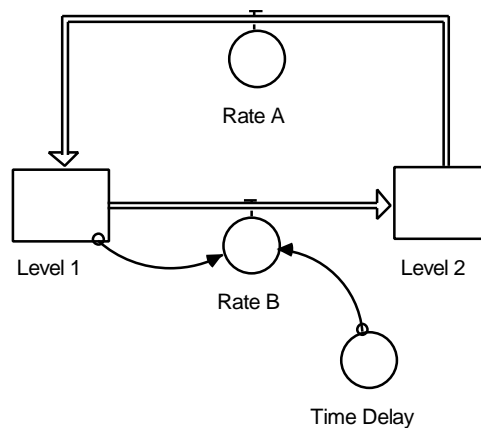
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<sup>8</sup> To initialize the model in equilibrium, first find the equilibrium values of the two levels, “Business Structures” and “Population,” from the base run of the model. Then enter the equilibrium values as initial values of the respective levels. The model should now remain in equilibrium during the entire simulation.

## 4. EXERCISE 2: MENTAL SIMULATION

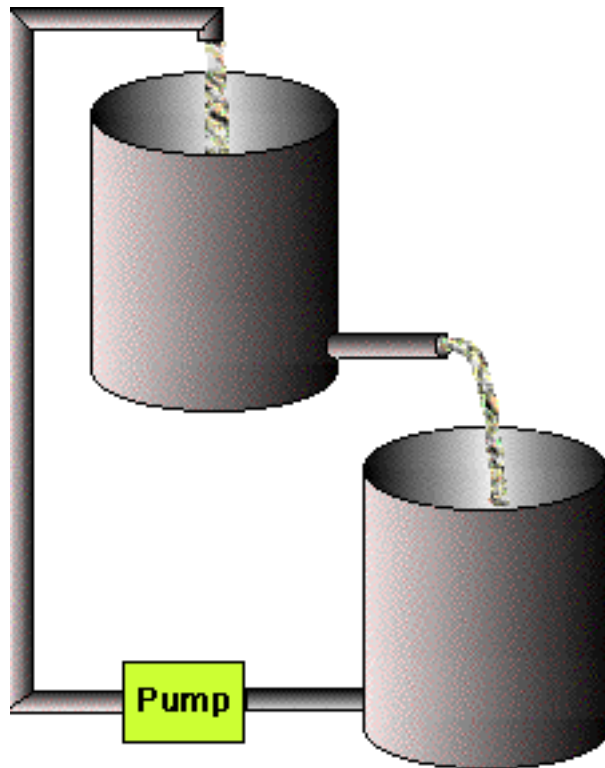
Mental simulation is a key tool of system dynamics. It is often tempting to run a model immediately after building it on the computer. It is equally tempting to trust the results from the first simulation run and assume that the modeler has adequately explored the system. In order for system dynamics modeling to be a useful tool, the modeler must be able to mentally simulate models. Mental simulation forces the modeler to thoroughly understand the reasons behind the model's behavior, and helps find problems of computer models.

In the following exercise, the reader will use the model shown in Figure 1 to perform mental simulation.



**Figure 1: A simple model used for practicing mental simulation skills**

The model in Figure 1 is analogous to the physical system shown in Figure 2.



**Figure 2: The physical equivalent**

In the physical system, the level of water in the containers corresponds to “Level 1” and “Level 2.” The pump corresponds to a constant “Rate A.” “Rate B” represents the spigot on the first container that flows at a rate proportional to the level of fluid in the upper container (“Level 1”).

The model is set up with the following conditions:

- ? “Level 1” is initially 0 gallons
- ? “Level 2” is initially 4 gallons
- ? The “Time Delay” is equal to 2 minutes
- ? “Rate A” is equal to 1 gallon per minute
- ? “Rate B,” in gallons per minute, is defined by:

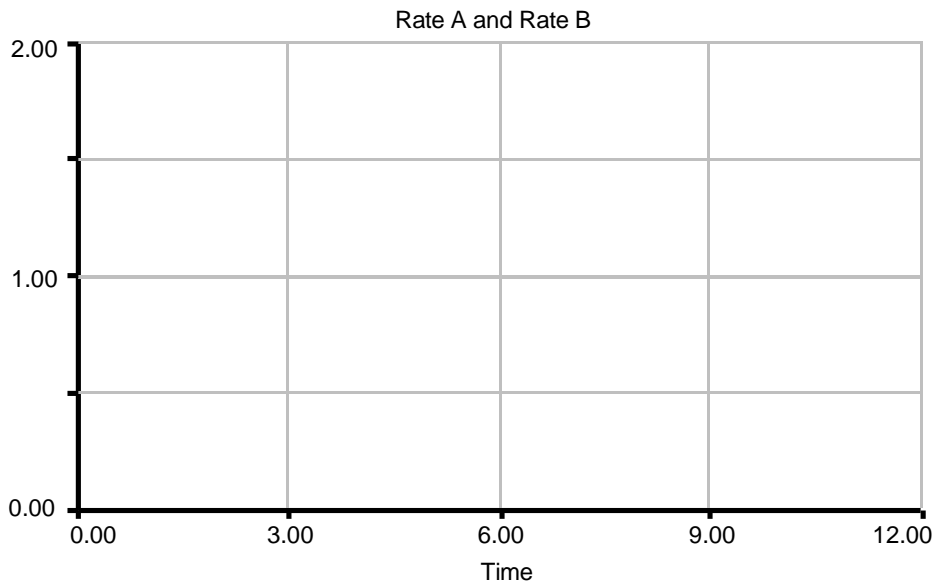
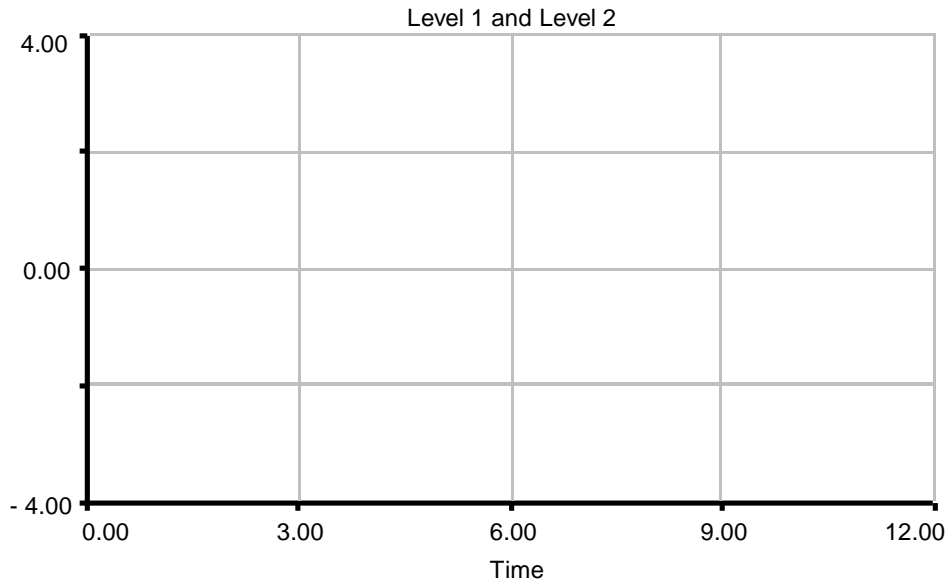
$$\text{Rate B} = \frac{\text{Level 1}}{\text{Time Delay}}$$

#### 4.1 Exploration of the Mental Simulation Model

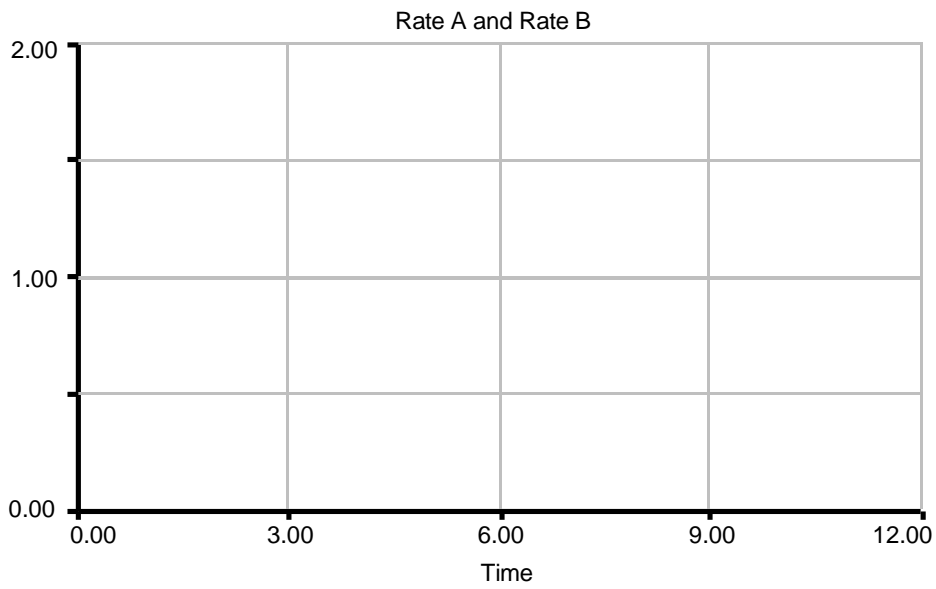
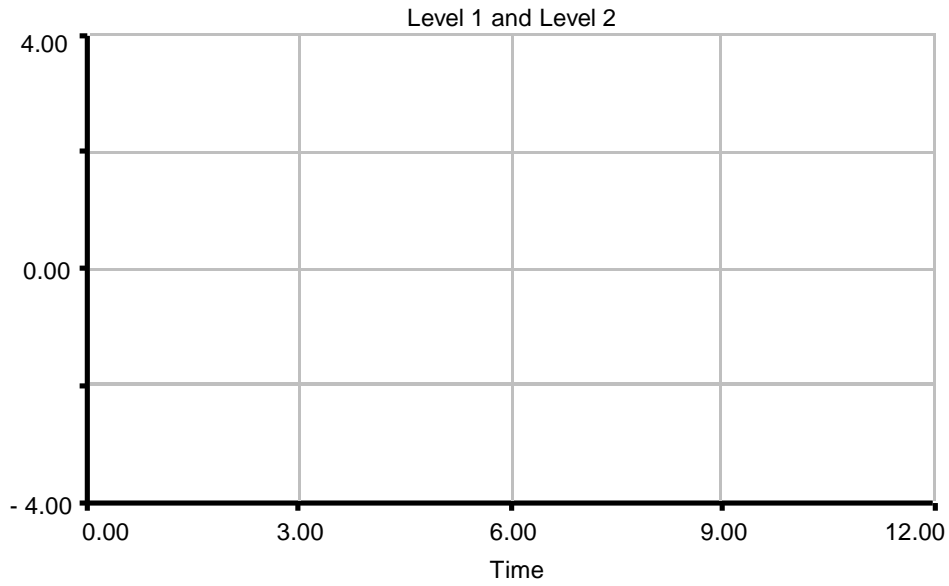
In the following exercises the reader will mentally simulate the model under three different conditions. Include as much information relating to the behavior of the model as possible. Only simple calculations are necessary. Some things that the reader might want to include are:

- ? initial and final value for “Level 1” and “Level 2”
- ? initial slope of “Level 1” and “Level 2”
- ? time constant of exponential behavior
- ? time-scale over which behavior occurs

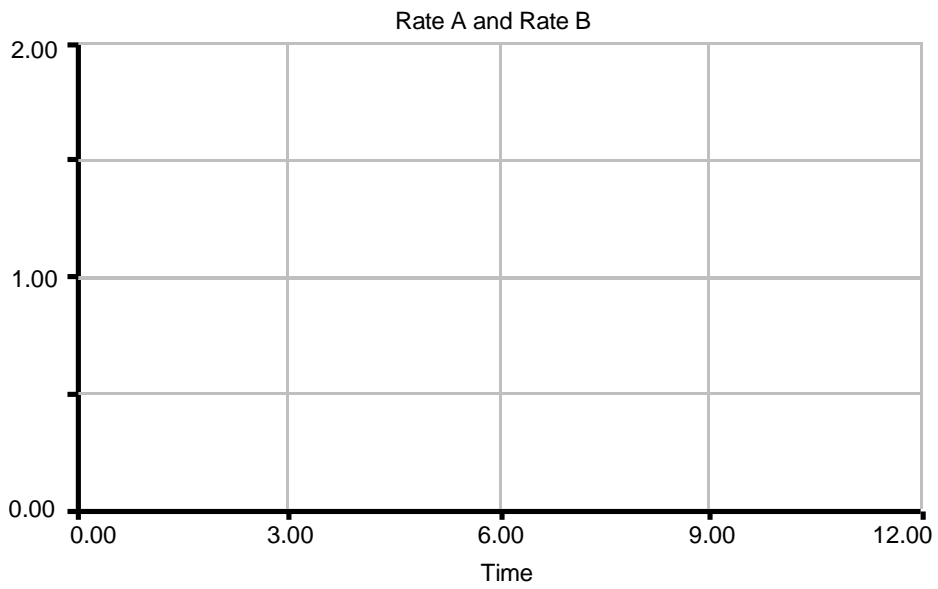
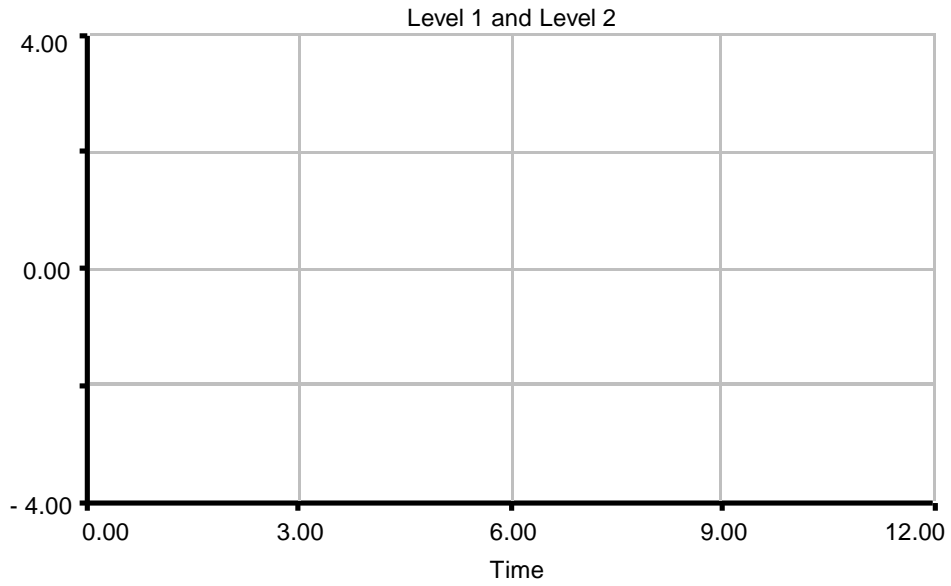
1. Mentally simulate the model with the initial conditions described in the previous section (“Level 1” = 0; “Level 2” = 4). Sketch the behavior of all model components on the blank graphs. Include as much information as possible.



2. Mentally simulate the same scenario (“Level 1” = 0; “Level 2” = 4). This time, change the value of “Time Delay” to 4.



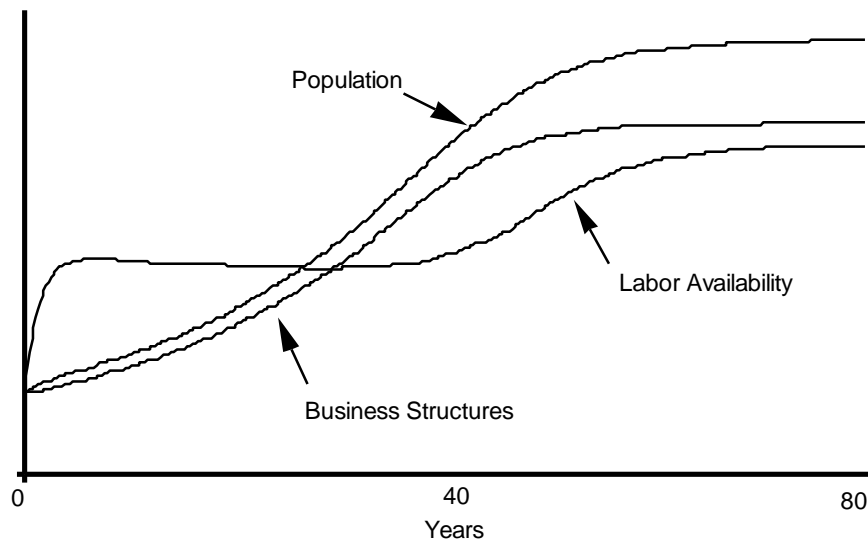
3. Now, assume that the “Time Delay” is equal to 2 again, but “Level 1” is initialized at 4 and “Level 2” at 0. Sketch what happens.



## 5. SOLUTION TO URBAN DYNAMICS EXERCISE

The following is one possible set of solutions to the questions posed in the exercise on urban growth.

1. Problem Statement: The rapid early growth of “Population” and “Business Structures” of an urban system ultimately gives way to a subsequent stagnant equilibrium characterized by a tight, static job market. After the initial growth, the total number of job opportunities per worker declines slowly, as “Business Structures” and “Population” move toward equilibrium. The reason for building the urban growth model is to explain the characteristic behavior of S-shaped growth that the system exhibits. The key variables of the system are “Business Structures,” “Population,” and “Labor Availability.” The time horizon over which the behavior occurs is approximately 60 to 80 years. Figure 3 shows a graph of the expected behavior of the key variables. The sketch shows the transition to equilibrium with high “Labor Availability” (unemployment) that was described in Section 3.1.



**Figure 3: System behavior**

2. The “Construction” of “Business Structures” is encouraged when “Land Fraction Occupied” is low, but becomes constrained as the “Land Area” fills up. People are attracted to the city at first when the “Job Attractiveness Multiplier” is high. As “Population” increases, “Labor Availability” increases to keep pace with business expansion, thereby encouraging further development. The success of previous investments also attracts more development. Eventually, the increased “Land Fraction Occupied” decreases the “Land Availability Multiplier,” and thus causes “Construction” to fall. The employment situation deteriorates because business “Construction” no longer keeps pace with demand for jobs. The city stagnates with a job shortage and most of the “Land Area” is occupied. The job shortage offsets the inherent attractiveness of the area to potential migrants.

3. Figure 4 shows a STELLA model for the urban growth system. Detailed model equations are included in Section 7.1 of the Appendix.

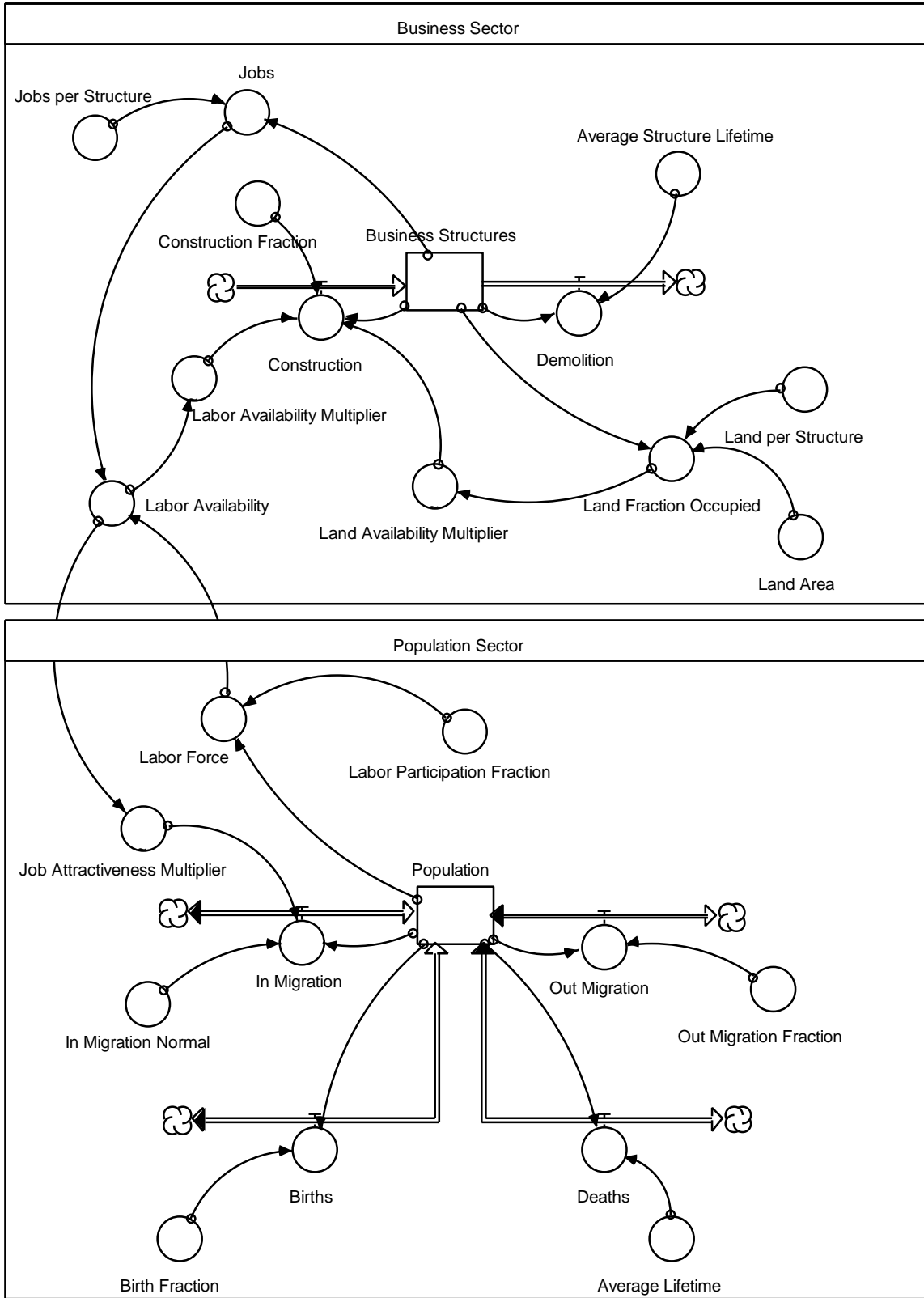


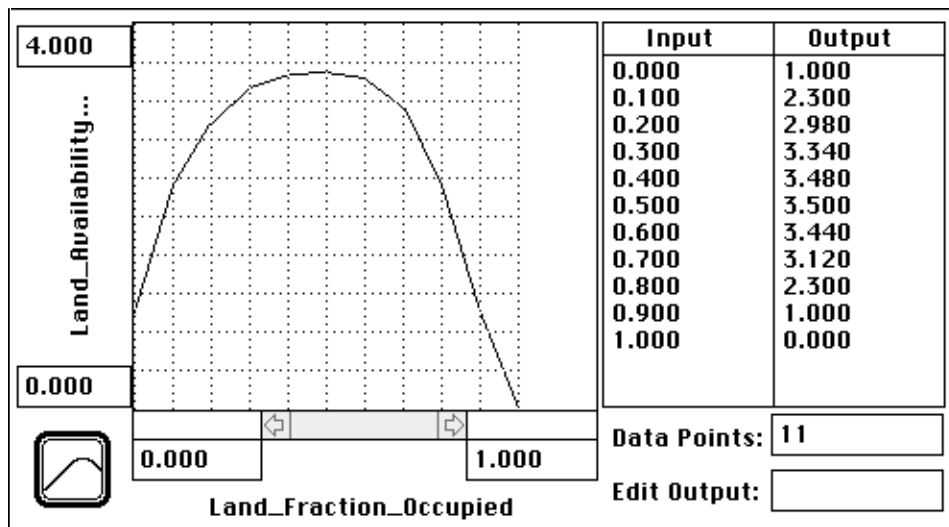
Figure 4: STELLA model of urban growth and stagnation

4. The model is a simple implementation of the description given in Section 3.4. The “Business Structures” sector consists of one stock, “Business Structures,” which is filled by “Construction” and drained by “Demolition.”

“Demolition” is defined by the familiar average lifetime formulation: the number of existing “Business Structures” divided by the “Average Structure Lifetime.”

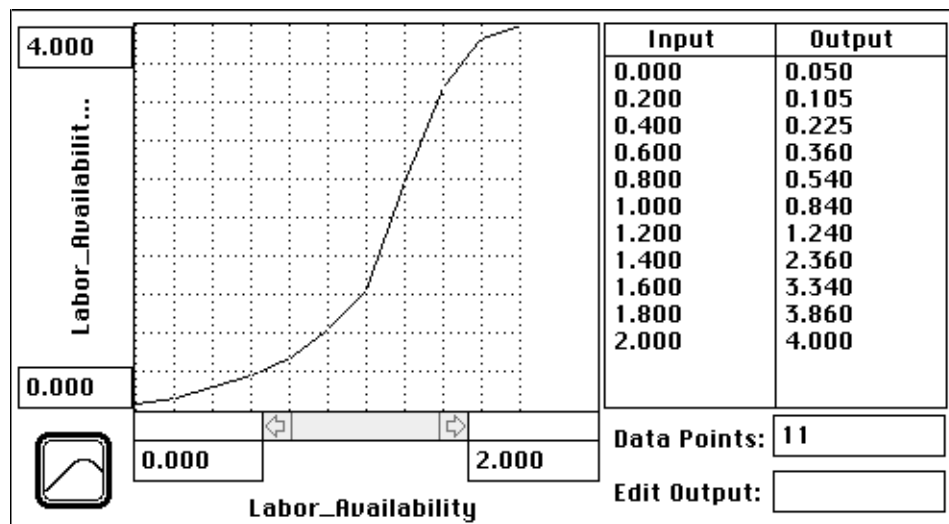
“Construction” is defined by the number of existing “Business Structures” multiplied by a “Construction Fraction” and two multipliers, the “Land Availability Multiplier” and the “Labor Availability Multiplier.”

The “Land Availability Multiplier” incorporates the effect of limited development “Land Area” on the “Construction” rate. It is determined by the “Land Fraction Occupied,” which is the amount of land occupied by existing “Business Structures” divided by the total “Land Area” available for commercial development. For low values of “Land Fraction Occupied,” increasing business development will lead to higher values for the “Land Availability Multiplier.” The high values occur because of the more advanced infrastructure associated with denser development. As the “Land Fraction Occupied” increases further, more desirable land is used up and land prices begin to rise, so “Construction” decreases. Eventually, when all the “Land Area” is used up and the “Land Fraction Occupied” is 1, “Construction” must fall to zero. The graphical function for the “Land Availability Multiplier” is shown in Figure 5.



**Figure 5: “Land Availability Multiplier” as a function of “Land Fraction Occupied”**

The “Labor Availability Multiplier” incorporates the effect of varying human resources on the “Construction” of new “Business Structures.” It is determined by the value of “Labor Availability,” the ratio of “Labor Force” to “Jobs.” When “Labor Availability” is low, there is little incentive to construct new “Business Structures” because employees are difficult to find. The “Labor Availability Multiplier” does not go all the way to zero because some “Construction” will still occur based on the investors’ assumption that the creation of “Jobs” will cause “In Migration.” As “Labor Availability” increases, so does the incentive to construct new “Business Structures.” Eventually, the “Labor Force” exceeds the number of “Jobs” to a great enough extent that the effects of high unemployment outweigh the advantages of high “Labor Availability,” so the “Labor Availability Multiplier” stops increasing and levels off. The graphical function for the “Labor Availability Multiplier” is shown in Figure 6.

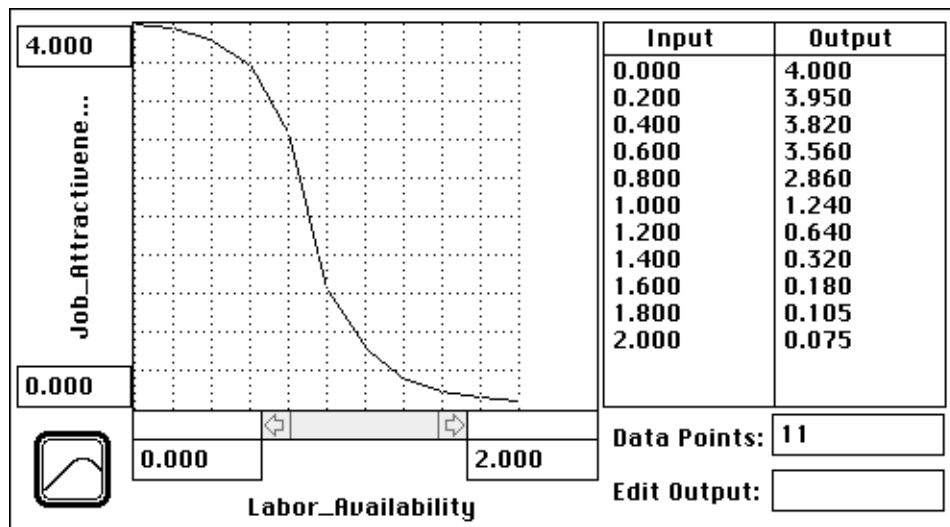


**Figure 6: “Labor Availability Multiplier” as a function of “Labor Availability”**

The “Population” sector contains the “Population” stock that is affected by “Births,” “Deaths,” “In Migration,” and “Out Migration.” “Births” are equal to 1.5% of the “Population” per year. “Deaths” are equal to the “Population” divided by the

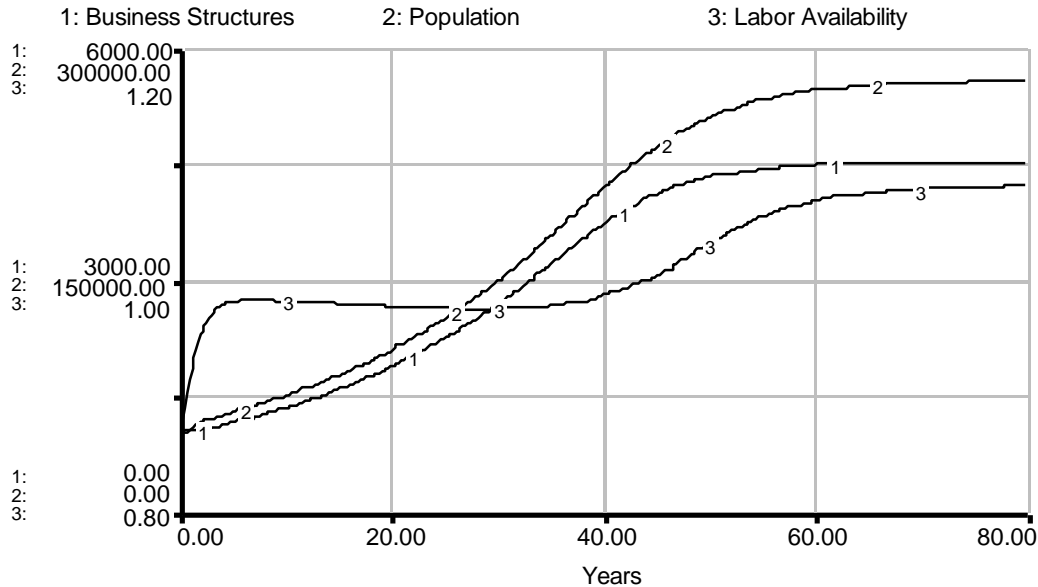
“Average Lifetime” of 67 years. “Out Migration” is assumed to be a constant 8% of the “Population” per year. “In Migration” is defined as the “Population” multiplied by the “In Migration Normal” of 8% per year and a “Job Attractiveness Multiplier.”

The “Job Attractiveness Multiplier” shows the effect of the “Labor Availability” on the number of people migrating into the urban area. When “Labor Availability” is low, the job market is very favorable to the working part of the “Population.” The number of “Jobs” exceeds the “Labor Force,” so good “Jobs” are relatively easy to come by. Naturally, people are attracted to move into the urban area, so the “Job Attractiveness Multiplier” is greater than 1. The curve is almost flat at the far left because it is assumed that when the conditions are already extremely favorable, small changes one way or the other will have little effect. As the “Labor Availability” increases, the employment conditions begin to decrease, so the tendency to migrate into the area declines. Eventually, when the conditions are extremely poor, the “Job Attractiveness Multiplier” falls to nearly zero. At this point, there is almost no “In Migration.” The graphical function of “Job Attractiveness Multiplier” is shown in Figure 7.



**Figure 7: “Job Attractiveness Multiplier” as a function of “Labor Availability”**

Figure 8 shows the base run of the urban growth model.



**Figure 8: Base run of the Urban Growth Model**

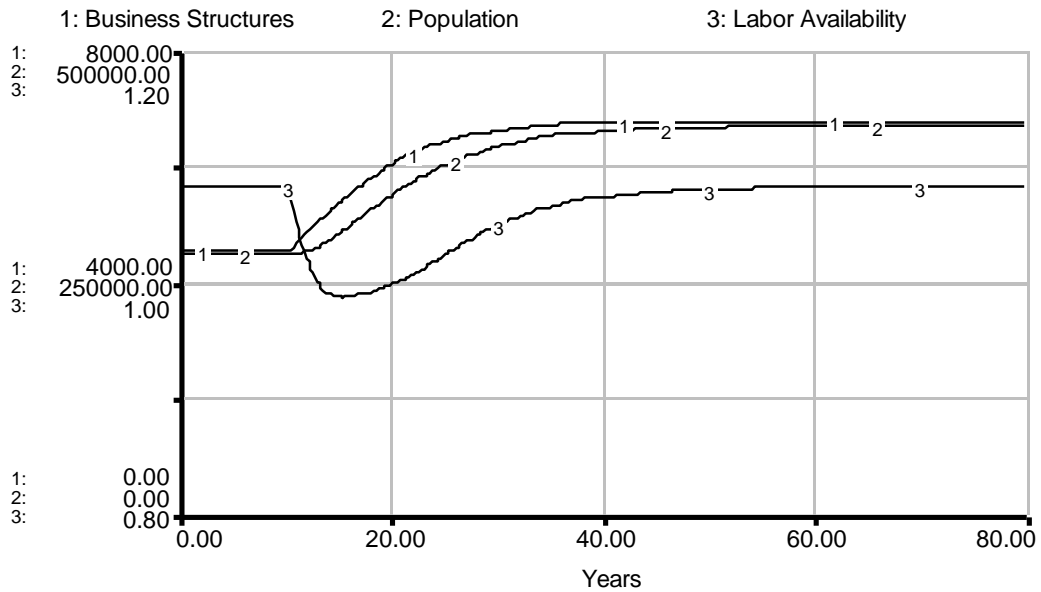
5. In the first portion of the model simulation, the city is attractive to residents and to people planning to move to the city because “Jobs” are readily available (“Labor Availability” is less than 1). The increasing “Population” and the success of previous investments demonstrate the potential for further growth and the profitability of “Construction” of “Business Structures.” The development of the urban infrastructure caused by business expansion permits further development to occur. The continued availability of “Land Area” during the growth stage allows business “Construction” to keep pace with the increasing “Population” so people continue to be attracted by the job opportunities in the area. The hypothesis stated in the answer to question 2 correctly identifies the reasons for the initial growth.

6. The quick initial increase in “Labor Availability” is due to the initial values of “Population” and “Business Structures” that are “out of balance” with one another. After the initial growth phase of “Business Structures” and “Population,” during which “Labor Availability” declines slightly (in other words, job availability improves slightly), “Jobs” become less available to the average job-seeker (“Labor Availability” starts increasing). The lower availability of “Jobs” occurs because “Jobs” begin rising slower than “Population” once the “Land Area” constraint slows business “Construction.” The increase in “Labor Availability” (corresponding to a decline in job availability) continues

until a sufficiently large unemployment rate is generated to discourage further “In Migration.” As noted earlier, in equilibrium, a job shortage must exist in the system in order to offset the inherent attractiveness of the area.

7. The constraint of a fixed “Land Area” limits growth by making additional “Construction” of “Business Structures” less and less economically justifiable. The model reaches equilibrium when the “Land Availability Multiplier” equals 1 and thus reduces “Construction” until it is equal to “Demolition.” Because “Construction” must be greater than zero, the “Land Fraction Occupied” must be less than 1 in equilibrium. In the base run, 90% of the “Land Area” is occupied in equilibrium.

Figure 9 shows the effect of annexing 50% additional “Land Area” to allow further urban expansion. This is a common policy used to counter the effects of urban stagnation. The model was initialized in equilibrium with the final equilibrium values from the base run. In year 10, the available “Land Area” was raised from 5000 to 7500 acres. Figure 9 shows that expanding the “Land Area” available creates a new expansion of “Business Structures” and “Population.” The expansion causes employment conditions to improve, but the relief is only temporary and the same problem of high “Labor Availability” in equilibrium exists. Again only 90% of the “Land Area” is occupied in equilibrium.



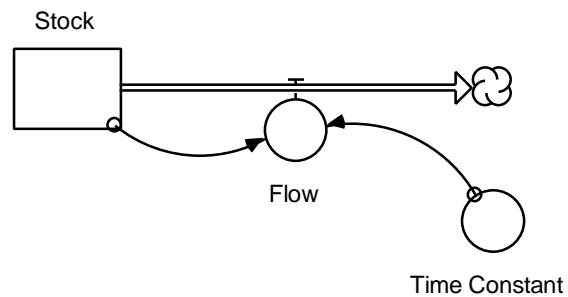
**Figure 9: Equilibrium for 50% step increase in available “Land Area”**

8. Assuming a fixed “Land Area” simplifies real life. Cities do expand their geographical boundaries, sometimes dramatically (consider Los Angeles and Mexico City). When total “Land Area” available is expanded in the simple two-level model, “Business Structures” and “Population” grow to fill the larger “Land Area.” However, the system still equilibrates with high “Labor Availability” (i.e., unemployment). If a series of expansions of the “Land Area” available is tested, each expansion will stimulate a further growth spurt. Each growth spurt will lower “Labor Availability” in the short term, due to increased business “Construction.” However, as each growth period gives way to a new equilibrium, “Labor Availability” will rise again to its previous equilibrium value. Therefore, expanding the city's geographic limits will lead to a temporary improvement in the availability of “Jobs,” but the improvement will eventually be reversed as the new “Land Area” is filled. The high unemployment equilibrium can only be avoided if the city continues to grow indefinitely. Because no city can expand indefinitely, analyzing growth within a fixed “Land Area” becomes a useful approach to studying the types of pressures that can limit growth.

## 6. SOLUTION TO MENTAL SIMULATION EXERCISE

### 6.1 A Note on Time Constants

An understanding of time constants is useful in mental simulation of models. A time constant is a number that relates a flow to the stock it affects. The definition of a time constant is the reciprocal of the number which, multiplied by the level, defines the rate. The model in Figure 11 shows one of the simplest examples of a time constant.

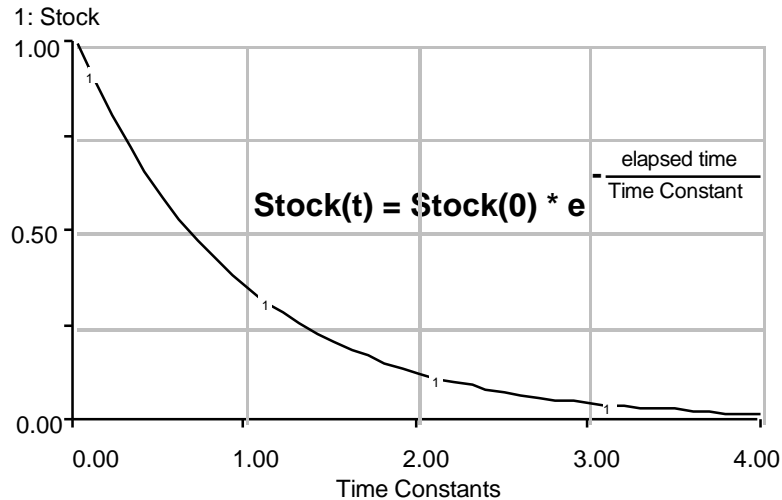


**Figure 11: A simple model using a time constant**

The rate equation in the model in Figure 11 is:

$$\text{Flow} = \frac{\text{Stock}}{\text{Time Constant}}$$

The model contains one negative feedback loop. The “Time Constant” affects the “Flow” that drains the “Stock.” The structure gives the following behavior for the “Stock”:



**Figure 12: Behavior of a single negative loop<sup>9</sup>**

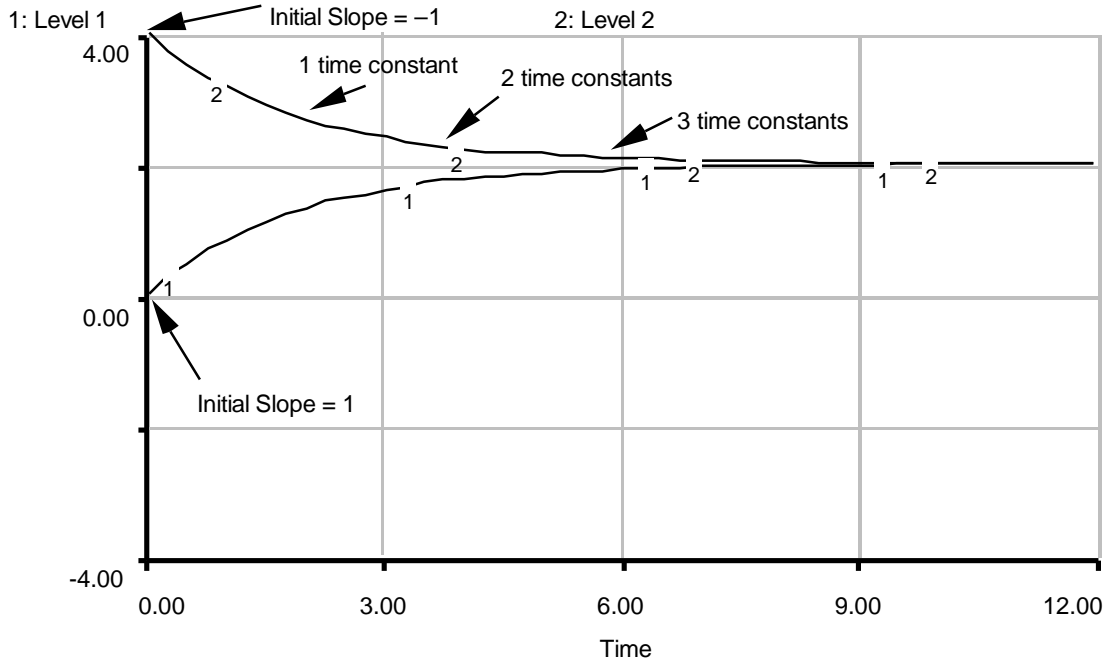
## 6.2 Mental Simulation Exercise Solution

1. Initial model run: Figures 13 and 14 show the behavior of the model. Both levels converge asymptotically to an equilibrium value that can be found by setting the net rate at each level equal to zero. Equilibrium occurs when “Rate A” equals “Rate B.” When “Level 1”/“Time Constant” equals 1, the system will be in equilibrium, and the equilibrium value for “Level 1” is 2. “Level 2” is affected by the same rates as “Level 1,” but in the opposite direction. As a result, any change in “Level 1” is mirrored by “Level 2.” Because “Level 1” increased by 2, “Level 2” decreased by 2 to a final value of 2.

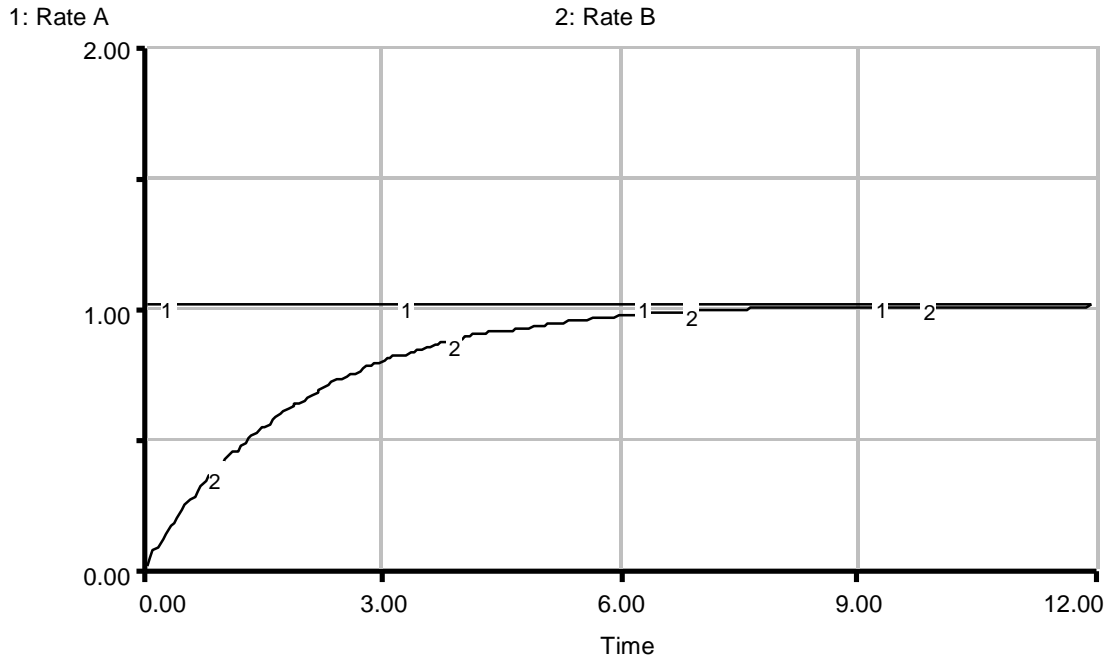
The slope of the graph of a level is simply the net rate entering the level. At the beginning of the simulation, “Level 1” is empty, so “Rate B” is 0. Therefore the only non-zero rate is “Rate A” which equals 1. The initial slope of “Level 1” is +1 and the initial slope of “Level 2” is -1.

The rest of the behavior can be determined using the “Time Constant” of the system, which is 2. The shape of the curve can be determined accurately with the chart and description outlined above. In Figure 13, the first three time constants are labeled.

<sup>9</sup> Mathematically inclined readers may realize that the negative sign in the exponent causes asymptotic growth. If the exponent is positive, growth is exponential.

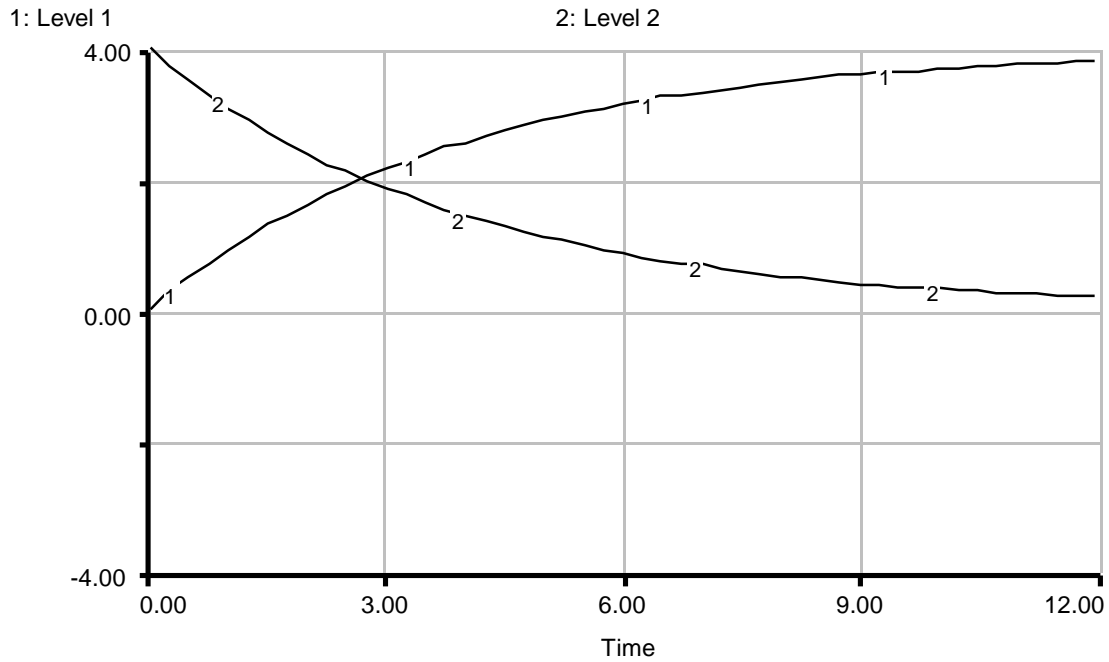


**Figure 13: First run behavior of “Level 1” and “Level 2”**

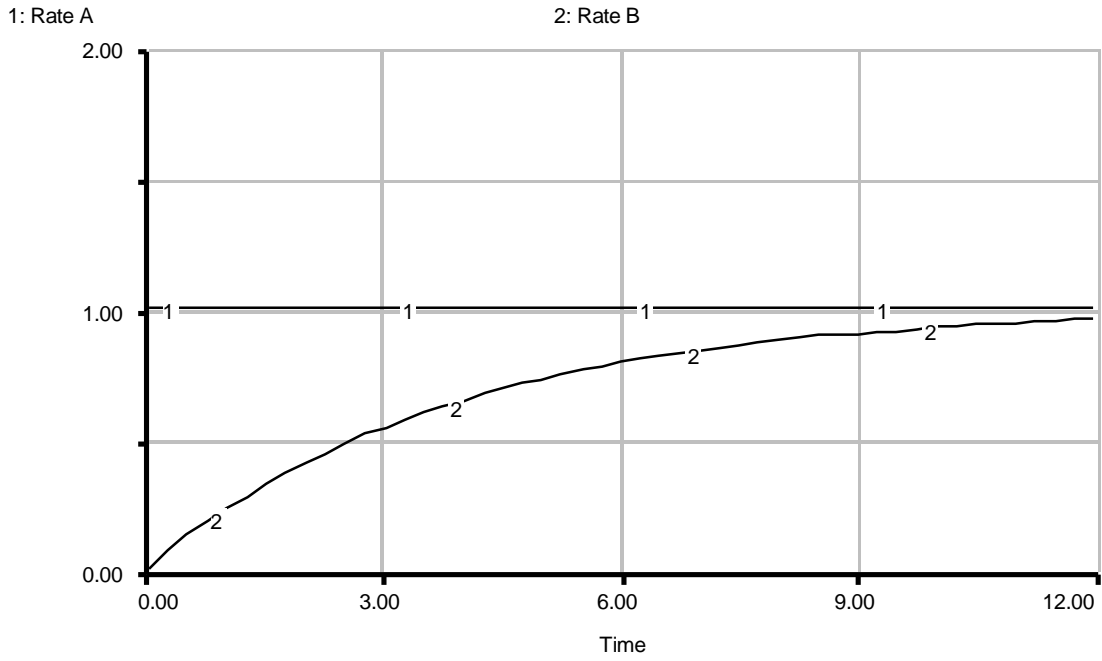


**Figure 14: First run behavior of “Rate A” and “Rate B”**

2. Changing the “Time Delay”: Figures 15 and 16 show the behavior of the second run. For the second run, the “Time Delay” was changed from 2 to 4. As a result, the equilibrium value for “Level 1” moved from 2 to 4. In addition, the approach to equilibrium was half the speed because the time constant was twice as large. As before, the change in “Level 2” mirrored “Level 1,” so “Level 2” went from 4 to 0.

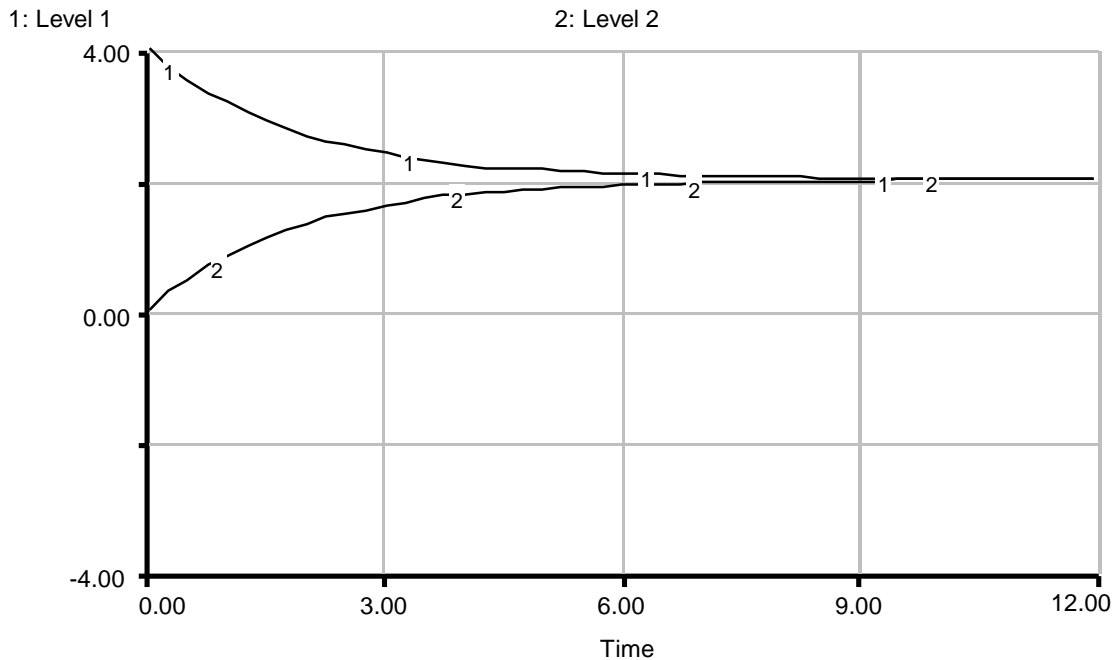


**Figure 15: Second run behavior of “Level 1” and “Level 2”**

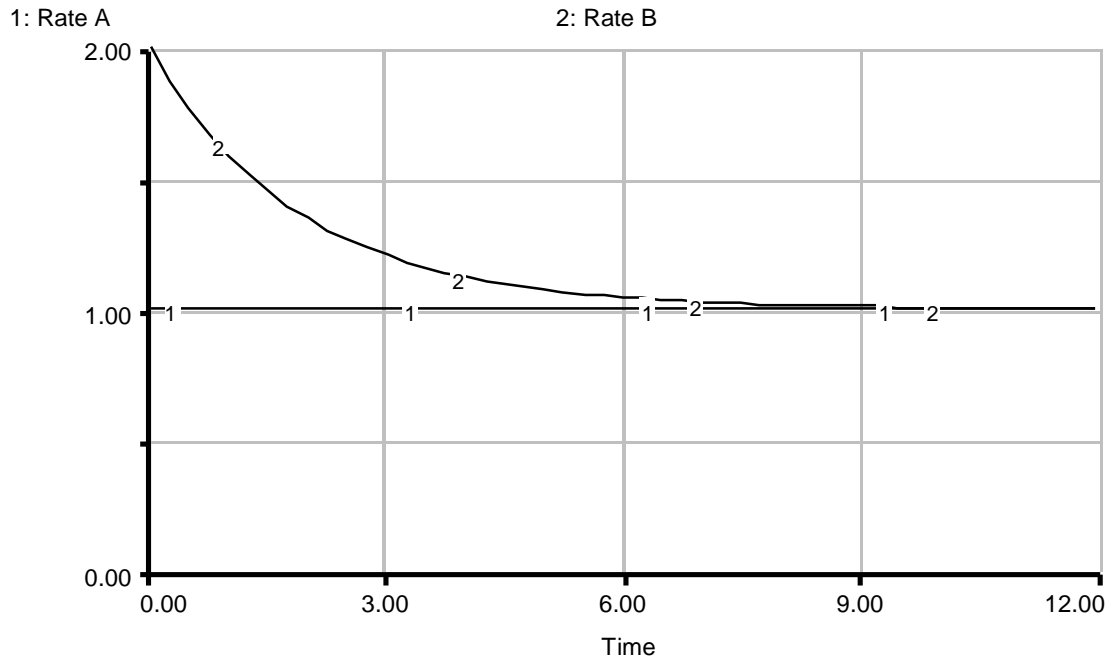


**Figure 16: Second run behavior of “Rate A” and “Rate B”**

3. Changing initial conditions: Figures 17 and 18 show the behavior of the model when the initial value of “Level 1” is changed from 0 to 4, and “Level 2” is initialized at 0. For the third model run, the time constant and equilibrium value for “Level 1” are the same as in the first run. However, because the initial value of “Level 1” is different, the model takes a different path to equilibrium. The first noticeable changes are the initial slopes for the levels. Because “Rate B” is not initially 0, but rather 2, the initial slopes of “Level 1” and “Level 2” are  $-1$  and  $+1$  respectively (the reverse of the first run). Again, the behavior of “Level 2” mirrors that of “Level 1,” so “Level 2” increases from 0 to 2.



**Figure 17: Third run behavior of “Level 1” and “Level 2”**



**Figure 18: Third run behavior of “Rate A” and “Rate B”**

## 7. APPENDIX

### 7.1 Equations for Urban Growth Model

#### Business Sector

$$\square \text{Business\_Structures}(t) = \text{Business\_Structures}(t - dt) + (\text{Construction} - \text{Demolition}) * dt$$

INIT Business\_Structures = 1000

DOCUMENT: Number of business structures in the city.

Units: structures

#### ☞ INFLOWS:

$$\text{Construction} = \text{Business\_Structures} * \text{Construction\_Fraction} * \text{Labor\_Availability\_Multiplier} *$$

Land\_Availability\_Multiplier

DOCUMENT: Rate of construction of business structures. It is affected by the number of already existing business structures, a normal construction fraction, and the availability of land and labor.

Units: structures/year

#### ☞ OUTFLOWS:

$$\text{Demolition} = \text{Business\_Structures} / \text{Average\_Structure\_Lifetime}$$

DOCUMENT: Rate of demolition of business structures.

Units: structures/year

$$\circ \text{Average\_Structure\_Lifetime} = 50$$

DOCUMENT: Average lifetime of a business structure.

Units: years

$$\circ \text{Construction\_Fraction} = 0.02$$

DOCUMENT: Normal rate of construction of business structure per existing business structure.

Units: 1/year

$$\circ \text{Jobs} = \text{Business\_Structures} * \text{Jobs\_per\_Structure}$$

DOCUMENT: Number of jobs provided by the existing business structures. It is the product of the number of business structures and the average number of jobs per structure.

Units: jobs

$$\circ \text{Jobs\_per\_Structure} = 20$$

DOCUMENT: The number of jobs provided by each structure.

Units: jobs/structure

$$\circ \text{Labor\_Availability} = \text{Labor\_Force} / \text{Jobs}$$

DOCUMENT: The ratio between the labor force and the number of available jobs.

Units: people/job

○ **Land\_Area** = 5000

DOCUMENT: Total land area available for commercial development.

Units: acres

○ **Land\_Fraction\_Occupied** =  $\text{Business\_Structures} * \text{Land\_per\_Structure} / \text{Land\_Area}$

DOCUMENT: The fraction of the commercial land that has already been developed.

Units: dimensionless

○ **Land\_per\_Structure** = 1

DOCUMENT: The amount of land required by each business structure.

Units: acres/structure

⊗ **Labor\_Availability\_Multiplier** = GRAPH(Labor\_Availability)

(0.00, 0.05), (0.2, 0.105), (0.4, 0.225), (0.6, 0.36), (0.8, 0.54), (1.00, 0.84), (1.20, 1.24), (1.40, 2.36), (1.60, 3.34), (1.80, 3.86), (2.00, 4.00)

DOCUMENT: The multiplier shows the effect of Labor Availability on Construction. When Labor Availability is greater than 1, businesses are more likely to construct new structures.

Units: dimensionless

⊗ **Land\_Availability\_Multiplier** = GRAPH(Land\_Fraction\_Occupied)

(0.00, 1.00), (0.1, 2.30), (0.2, 2.98), (0.3, 3.34), (0.4, 3.48), (0.5, 3.50), (0.6, 3.44), (0.7, 3.12), (0.8, 2.30), (0.9, 1.00), (1, 0.00)

DOCUMENT: The multiplier shows the effect of availability of land on business construction. When nearly all the land is available, Construction is slow because there are very few businesses around to provide capital. As more of the Land Area is occupied, the effect on Construction increases. As the available Land Area is used up, Construction drops until it is zero at maximum capacity.

Units: dimensionless

### Population Sector

□ **Population(t)** =  $\text{Population}(t-dt) + (\text{In\_Migration} + \text{Births} - \text{Out\_Migration} - \text{Deaths}) * dt$

INIT Population = 50000

DOCUMENT: The number of people living in the urban area.

Units: people

⌘ INFLOWS:

$\text{In\_Migration} = \text{Population} * \text{In\_Migration\_Normal} * \text{Job\_Attractiveness\_Multiplier}$

DOCUMENT: The number of people who move into the urban area each year. It is affected by the current population, a normal fraction of in-migration, and the availability of jobs.

units: people/year

☞ Births = Population \* Birth\_Fraction

DOCUMENT: The number of people born in the area per year.

Units: people/year

☞ OUTFLOWS:

Out\_Migration = Population \* Out\_Migration\_Fraction

DOCUMENT: The number of people who leave the urban area each year.

Units: people/year

☞ Deaths = Population / Average\_Lifetime

DOCUMENT: Number of people who die each year.

Units: people/year

○ **Average\_Lifetime** = 66.7

DOCUMENT: The average lifetime of a person living in the urban area is approximately 67 years.

Units: years

○ **Birth\_Fraction** = 0.015

DOCUMENT: The fraction of the population that reproduces each year.

Units: 1/year

○ **In\_Migration\_Normal** = 0.08

DOCUMENT: The fraction of the population that immigrates each year under normal conditions.

Units: 1/year

○ **Labor\_Force** = Population \* Labor\_Participation\_Fraction

DOCUMENT: The number of people who are eligible to work. It is a constant fraction of the population.

Units: people

○ **Labor\_Participation\_Fraction** = 0.35

DOCUMENT: The fraction of the total population that is willing and able to work.

Units: dimensionless

○ **Out\_Migration\_Fraction** = 0.08

DOCUMENT: The fraction of the population that emigrates each year.

Units: 1/year

- ⊗ **Job\_Attractiveness\_Multiplier** = GRAPH(Labor\_Availability)  
 (0.00, 4.00), (0.2, 3.95), (0.4, 3.82), (0.6, 3.56), (0.8, 2.86), (1.00, 1.24), (1.20, 0.64), (1.40, 0.32), (1.60, 0.18), (1.80, 0.105), (2.00, 0.075)  
 DOCUMENT: The multiplier shows the effect of Labor Availability on immigration. When there are many available jobs (Labor Availability is less than 1), people are inclined to move to the city. When there are not enough available jobs (Labor Availability is greater than 1), people tend not to immigrate to the urban area.  
 Units: dimensionless

## 7.2 Equations for Mental Simulation Model

- **Level\_1(t)** = Level\_1(t - dt) + (Rate\_A - Rate\_B) \* dt  
 INIT Level\_1 = 4  
 Units: gallons

⌘ INFLOWS:  
 Rate\_A = 1  
 Units: gallons/minute

⌘ OUTFLOWS:  
 Rate\_B = Level\_1/Time\_Delay  
 Units: gallons/minute

- **Level\_2(t)** = Level\_2(t - dt) + (Rate\_B - Rate\_A) \* dt  
 INIT Level\_2 = 0  
 Units: gallons

⌘ INFLOWS:  
 Rate\_B = Level\_1/Time\_Delay  
 Units: gallons/minute

⌘ OUTFLOWS:  
 Rate\_A = 1  
 Units: gallons/minute

- **Time\_Delay** = 2  
 Units: minutes

# Vensim Examples: Modeling Exercises

## Section 2

Aaron Diamond & Nathaniel Choge  
October 2001

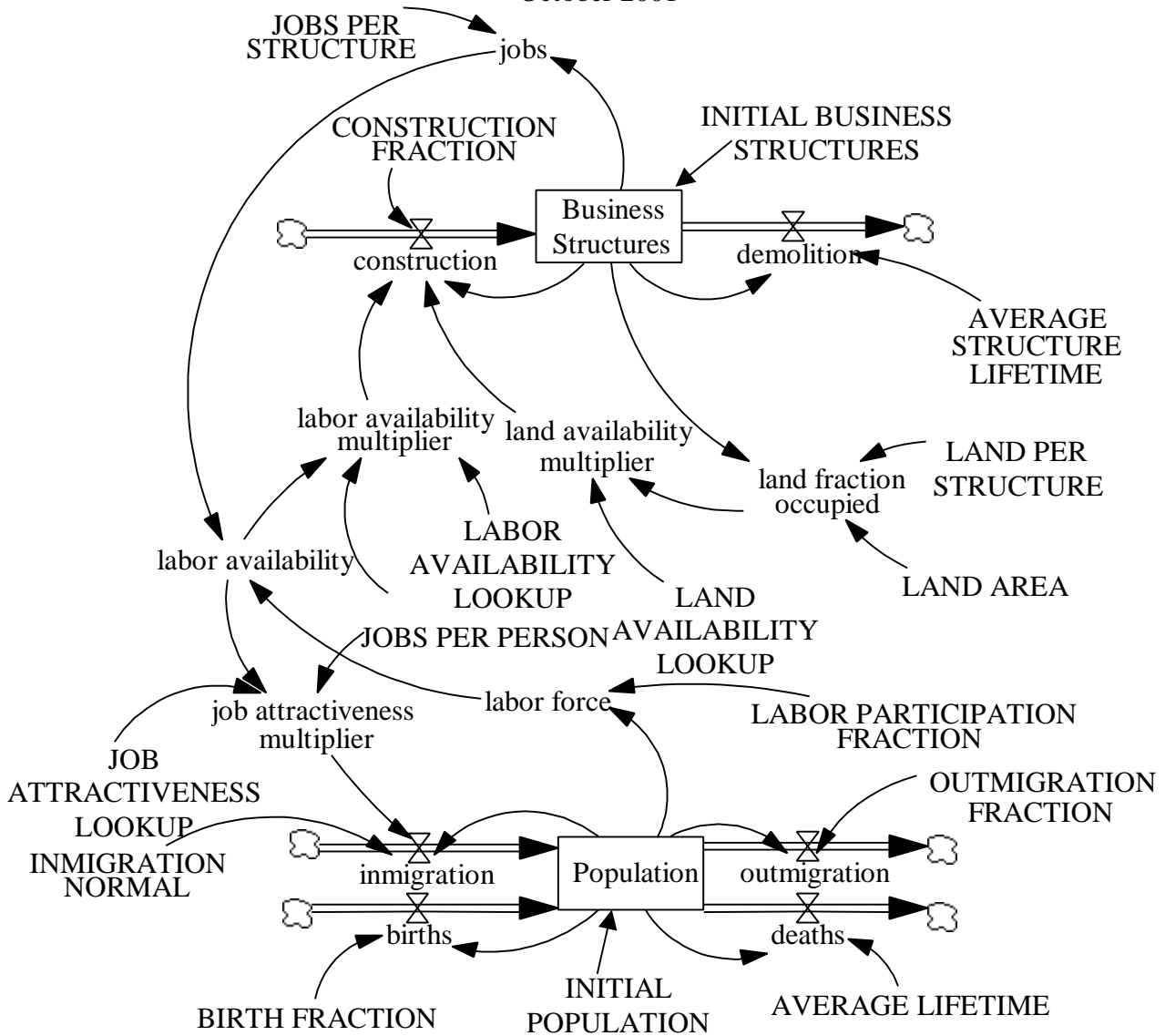


Figure 19: Vensim equivalent of Figure 4: model of urban growth and stagnation

**Documentation for the Urban Growth Model:**

(01) AVERAGE LIFETIME=66.7

Units: year

The average lifetime of a person living in the urban area is approximately 66.7 years.

(02) AVERAGE STRUCTURE LIFETIME=50

Units: year

Average lifetime of a business structure.

(03) BIRTH FRACTION=0.015

Units: 1/year

The fraction of the Population that reproduces each year.

(04) births=Population\*BIRTH FRACTION

Units: people/year

The number of people born in the area per year.

(05) Business Structures= INTEG (construction-demolition, INITIAL BUSINESS STRUCTURES)

Units: structures

Number of business structures in the city.

(06) construction=Business Structures\*CONSTRUCTION FRACTION\*labor availability multiplier\*land availability multiplier

Units: structures/year

Rate of construction of business structures. It is affected by the number of already existing business structures, a normal construction fraction, and the availability of land and labor.

- (07) CONSTRUCTION FRACTION=0.02  
Units: 1/year  
Normal rate of construction of business structure per existing business structure.
- (08) deaths=Population/AVERAGE LIFETIME  
Units: people/year  
Number of people who die each year.
- (09) demolition=Business Structures/AVERAGE STRUCTURE LIFETIME  
Units: structures/year  
Rate of demolition of business structures.
- (10) FINAL TIME = 80  
Units: year  
The final time for the simulation.
- (11) INITIAL BUSINESS STRUCTURES=1000  
Units: structures
- (12) INITIAL POPULATION=50000  
Units: people
- (13) INITIAL TIME = 0  
Units: year  
The initial time for the simulation.
- (14) immigration=Population\*INMIGRATION NORMAL\*job attractiveness multiplier  
Units: people/year  
The number of people who move into the urban area each year. It is affected by the current Population, a normal fraction of in-migration, and the availability of jobs.

- (15) INMIGRATION NORMAL=0.08  
 Units: 1/year  
 The number of people who are eligible to work. It is a constant fraction of the population.
- (16) JOB ATTRACTIVENESS LOOKUP=([(0,0)(10,10)],(0,4),(0.2,3.95),(0.4,3.82),  
 (0.6,3.56),(0.8,2.86),(1,1.24),(1.2,0.64),(1.4,0.32),(1.6,0.18),(1.8,0.105),(2,0.075)  
 )  
 Units: dmnl
- (17) job attractiveness multiplier=JOB ATTRACTIVENESS LOOKUP (labor  
 availability/JOBS PER PERSON)  
 Units: dmnl  
 The multiplier shows the effect of labor availability on immigration. When there are many available jobs (labor availability is less than 1), people are inclined to move to the city. When there are not enough available jobs (labor availability is greater than 1), people tend not to immigrate to the urban area.
- (18) jobs=Business Structures\*JOBS PER STRUCTURE  
 Units: jobs  
 Number of jobs provided by the existing structures. It is the product of the number of business structures and the average number of jobs per structure.
- (19) JOBS PER PERSON=1  
 Units: people/jobs
- (20) JOBS PER STRUCTURE=20

Units: jobs/structures

The number of jobs provided by each structure.

(21) labor availability=labor force/jobs

Units: people/jobs

The ratio between the labor force and the number of available jobs.

(22) LABOR AVAILABILITY LOOKUP([(0,0)-(10,10)],(0,0.05), (0.2,0.105),  
(0.4,0.225),(0.6,0.36),(0.8,0.54),(1,0.84),(1.2,1.24),(1.4,2.36),(1.6,3.34),(1.8,3.86  
) ,(2,4))

Units: dmnl

(23) labor availability multiplier=LABOR AVAILABILITY LOOKUP (labor  
availability/JOBS PER PERSON)

Units: dmnl

The multiplier shows the effect of labor availability on  
Construction. When labor availability is greater than 1,  
businesses are more likely to construct new structures.

(24) labor force=Population\*LABOR PARTICIPATION FRACTION

Units: people

The number of people who are eligible to work. It is a constant  
fraction of the population.

(25) LABOR PARTICIPATION FRACTION=0.35

Units: dmnl

The fraction of the total Population that is willing and able to  
work.

(26) LAND AREA=5000

Units: acres

The total land area available for commercial development.

- (27) LAND AVAILABILITY LOOKUP([(0,0)-(10,10)],(0,1),(0.1,2.3),(0.2,2.98),(0.3,3.34),(0.4,3.48),(0.5,3.5),(0.6,3.44),(0.7,3.12),(0.8,2.3),(0.9,1),(1,0))

Units: dmnl

The multiplier shows the effect of availability of land on business construction.

When nearly all the land is available, construction is slow because there are very few businesses

around to provide capital. As more of the LAND AREA is occupied, the effect on construction increases. As the available LAND AREA is used up, construction drops until it is zero at maximum capacity.

- (28) land availability multiplier=LAND AVAILABILITY LOOKUP (land fraction occupied)

Units: dmnl

- (29) land fraction occupied=Business Structures\*LAND PER STRUCTURE/LAND AREA

Units: dmnl

The fraction of the commercial land that has already been developed.

- (30) LAND PER STRUCTURE=1

Units: acres/structures

The amount of land required by each business structure.

- (31) outmigration=Population\*OUTMIGRATION FRACTION

Units: people/year

The number of people who leave the urban area each year.

(32) OUTMIGRATION FRACTION=0.08

Units: 1/year

The fraction of the Population that emigrates each year.

(33) Population= INTEG (births+immigration-deaths-outmigration, INITIAL POPULATION)

Units: people

The number of people living in the urban area.

(34) SAVEPER = TIME STEP

Units: year

The frequency with which output is stored.

(35) TIME STEP = 0.0625

Units: year

The time step for the simulation.

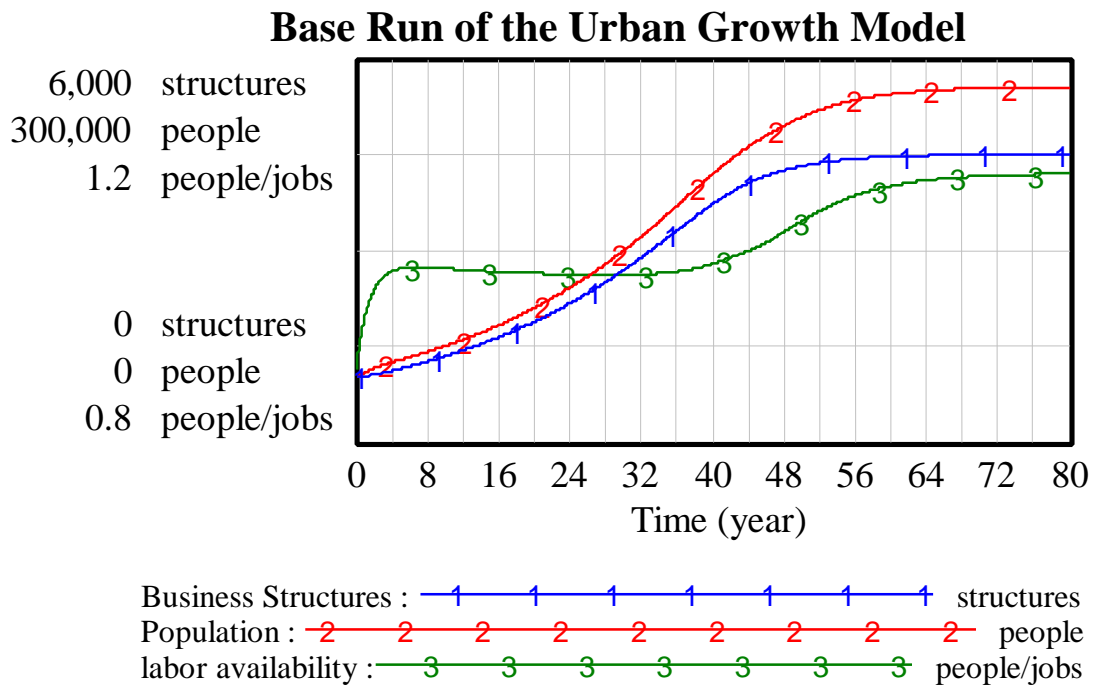


Figure 20: Vensim equivalent of Figure 8: Base run of the Urban Growth Model

### Model Run with STEP INCREASE in LAND AREA

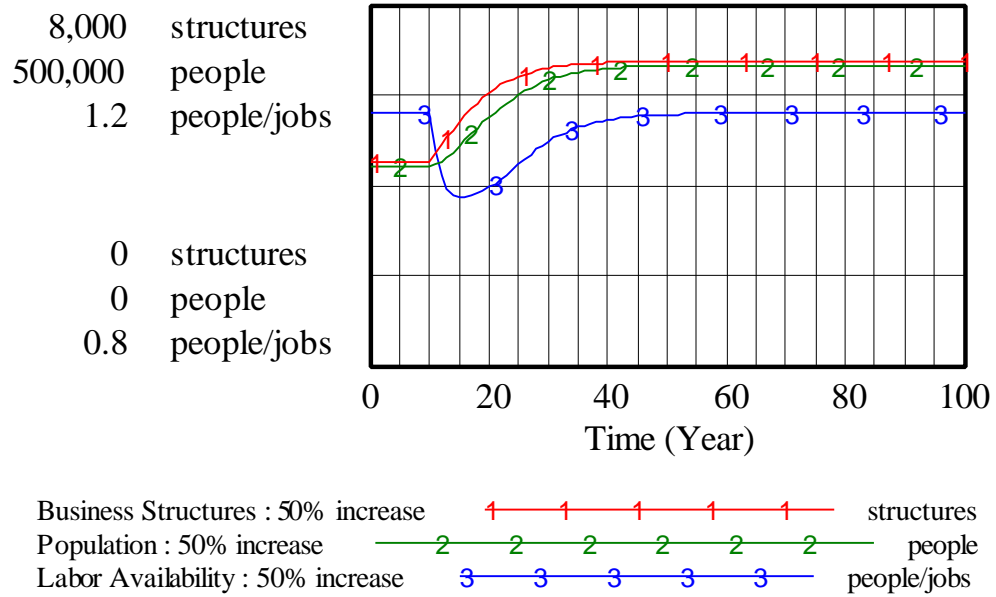


Figure 21: Vensim correction of Figure 9: Equilibrium for 50% step increase in available “LAND AREA”

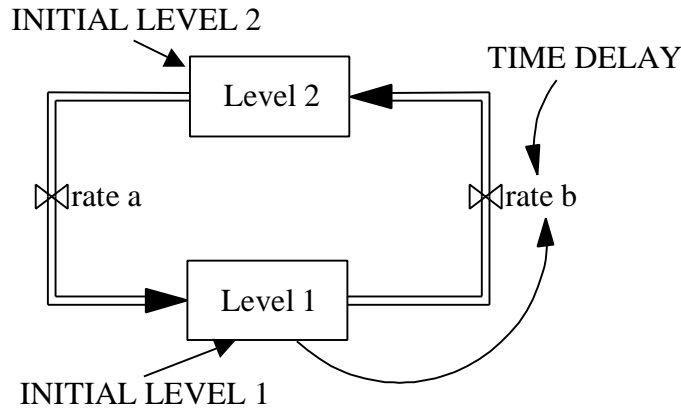


Figure 22: Vensim Equivalent of Figure 1: A simple model used for practicing mental simulation skills.

**Documentation for Mental Simulation Model:**

- (01) FINAL TIME = 12  
Units: minutes  
The final time for the simulation.
- (02) INITIAL LEVEL 1=4  
Units: gallons
- (03) INITIAL LEVEL 2=0  
Units: gallons
- (04) INITIAL TIME = 0  
Units: minutes  
The initial time for the simulation.
- (05) Level 1= INTEG (rate A-rate B,INITIAL LEVEL 1)  
Units: gallons
- (06) Level 2= INTEG (+rate B-rate A,INITIAL LEVEL 2)  
Units: gallons
- (07) rate a=1  
Units: gallons/minutes
- (08) rate b=Level 1/TIME DELAY  
Units: gallons/minutes
- (09) SAVEPER = TIME STEP

Units: minutes

The frequency with which output is stored.

(10) TIME DELAY=2

Units: minutes

(11) TIME STEP = 0.0625

Units: minutes

The time step for the simulation.

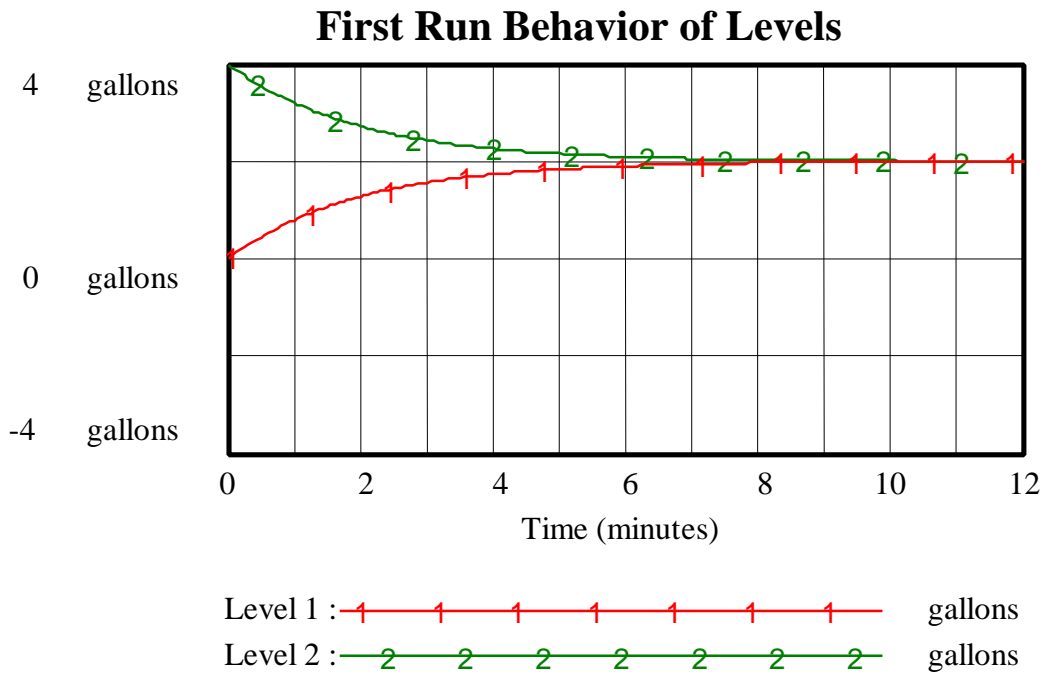


Figure 23: Vensim equivalent of Figure 13: First run behavior of “Level 1 and Level 2” (Level 1 is initially 0 and level 2 is initially 4 for the first and second runs)

### First Run Behavior of rates

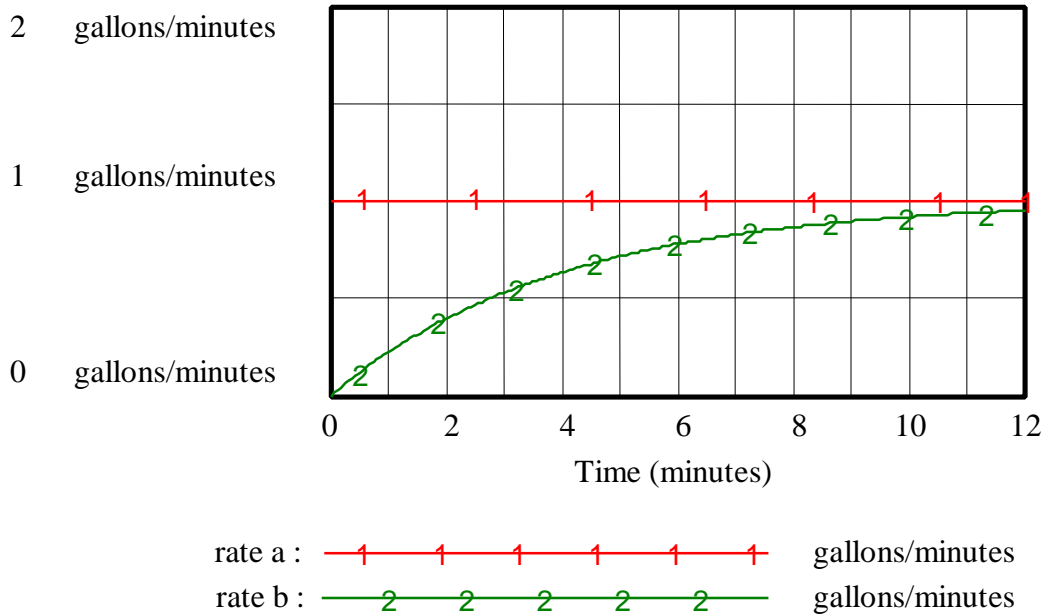


Figure 24: Vensim equivalent of Figure 14: First run behavior of “rate a” and “rate b”

### Second Run Behavior of Levels

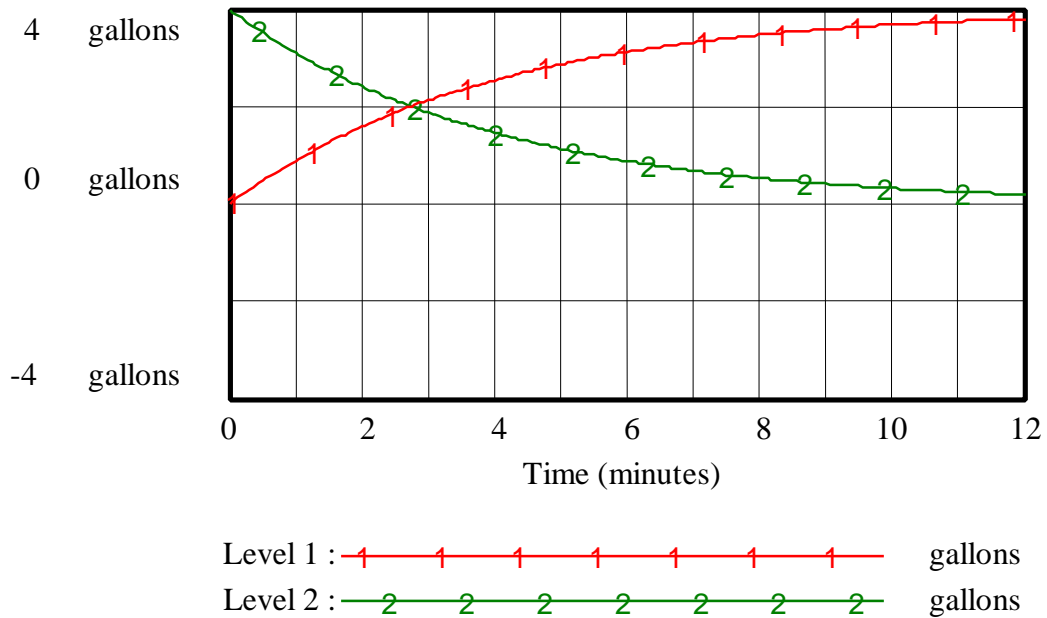


Figure 25: Vensim equivalent of Figure 15: Second run behavior of “Level 1” and “Level 2”





Figure 28: Vensim equivalent of Figure 18: Third run behavior of “rate a” and “rate b”